

# Advanced Computer Networks

## Congestion control

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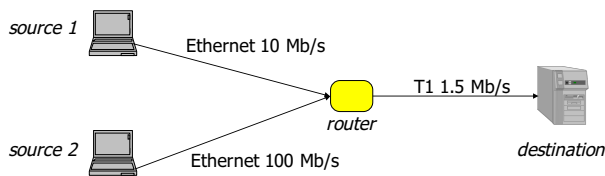
1

2

## Contents

- Objectives of Congestion Control
  - efficiency
  - fairness
- Max-min fairness
- Proportional fairness
- Additive increase, multiplicative decrease
- Different forms of congestion control

## Congestion control



- How to allocate network resources?
  - link capacity
  - buffers at routers or switches
- What to do when the traffic exceeds link capacity?
  - congestion control

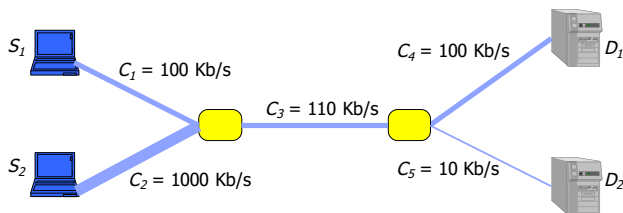
3

4

## Performance criteria

- Efficiency
  - best use of allocated resources
  - max throughput - 100 % utilization
  - min delay - 0 % utilization
- Fairness (équité)
  - fair share to each user
  - different definitions of fairness
    - equal share
    - max-min fairness
    - proportional fairness

## Congestion Control - example



- Sources send as much as possible
- Allocation of throughput
  - if the offered traffic exceeds the capacity of a link, all sources see their traffic reduced in proportion of their offered traffic
  - approximately true if FIFO in routers

5

6

## Throughput allocation

- Throughput  $x_{ls}$  : source  $s$  on link  $l$
- Traffic  $\lambda_s$  : generated by source  $s$
- Allocation

$$x_{11} = \min(\lambda_{1r}, C_1)$$

$$x_{22} = \min(\lambda_{2r}, C_2)$$

$$x_{3i} = \min(x_{ir}, C_3 x_{ii} / (x_{11} + x_{22}))$$

Our example:

$$x_{11} = 100$$

$$x_{22} = 1000$$

$$x_{31} = 110 \times 100 / 1100 = 10$$

$$x_{32} = 110 \times 1000 / 1100 = 100$$

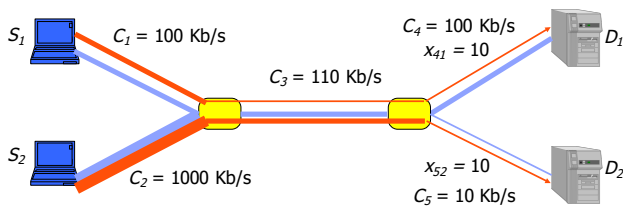
$$x_{41} = 10$$

$$x_{52} = 10$$

$$\text{throughput } \theta = x_{41} + x_{52}$$

$$\text{throughput } \theta = 20 \text{ Kb/s}$$

## Congestion Control - example



- S1 sends 10 Kb/s because it is competing with S2 on link 3
- S2 is limited on link 5 anyway

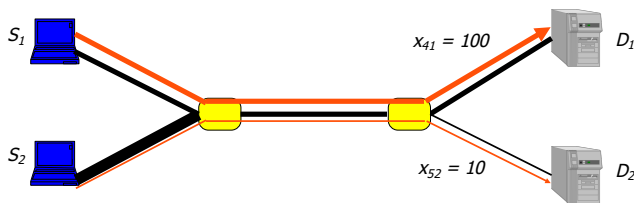
7

## Congestion Control - example

- How to increase throughput?
  - if  $S_2$  is aware of the global situation and if it would cooperate
  - $S_2$  reduces  $x_{52}$  to 10 Kb/s, because anyway, it cannot send more than 10 Kb/s on link 5
  - $x_{31} = 100$  Kb/s and  $x_{41} = 100$  Kb/s without any penalty for  $S_2$
  - throughput is now  $\theta = 110$  Kb/s

8

## Congestion Control - example



- Optimal use of resources

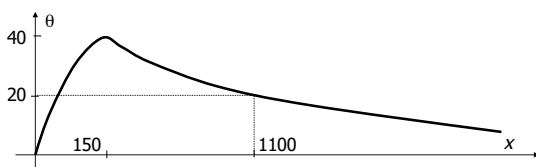
9

## Efficiency criterion

- In a packet network, sources should limit their sending rate by taking into consideration the state of the network. Ignoring this may put the network into congestion collapse
  - network resources are not used efficiently
  - performance indices perceived by sources are not satisfactory
- One objective of congestion control is to avoid such inefficiencies

10

## Throughput vs. offered load

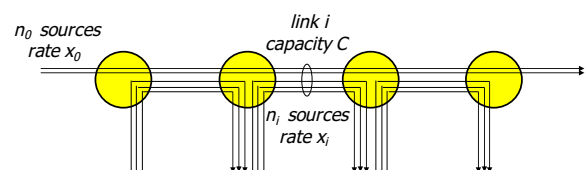


- Same example - sources increase their throughput in parallel but at different rate
  - $\lambda_1 = \lambda$ ,  $\lambda_2 = \lambda^2/10$ ,  $\lambda$  - a parameter
  - $\lambda_1(1) = 1$ ,  $\lambda_2(1) = 1/10$
  - $\lambda_1(10) = 10$ ,  $\lambda_2(10) = 10$
  - $\lambda_1(100) = 100$ ,  $\lambda_2(100) = 1000$
  - offered load  $x = \lambda_1 + \lambda_2$
  - $x = 1100$ ,  $\theta = 20$  Kb/s

11

## Efficiency versus Fairness

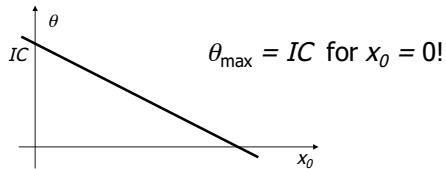
- Parking lot scenario
  - link capacity :  $C$
  - $n_i$  sources, rate  $x_i$ ,  $i = 1, \dots, I$
  - traffic on link  $i$  :  $n_0 x_0 + n_i x_i$



12

## Maximal throughput

- For given  $n_0$  and  $x_0$ , maximizing the throughput requires that
  - $n_i x_i = C - n_0 x_0$
- Total throughput, measured at the network output
  - $\theta = n_0 x_0 + \sum n_i x_i = n_0 x_0 + \sum (C - n_0 x_0) = n_0 x_0 + I(C - n_0 x_0) = IC - (I - 1) n_0 x_0$



13

## Fairness

- Maximizing network throughput as a primary objective may lead to large unfairness
  - some sources may get a zero throughput
- Fairness criterion
  - let allocate the same share to all sources, e.g. for  $n_i = 1$ 
    - $x_i = C/2$
    - $\theta_{fair} = (I+1)C/2$
  - roughly half of the maximal throughput

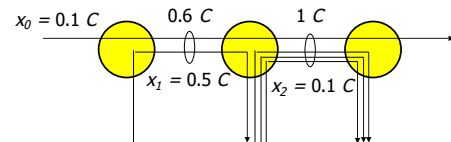
14

## Equal share fairness

- Consider the parking lot scenario for general values of  $n_i$ 
  - equal share on link  $i$ 
    - $x_i = C / (n_0 + n_i), i = 1, \dots, I$
  - let decrease  $x_0$  to increase  $\theta$  (we have seen that this maximizes throughput)
    - $x_0 = \min C / (n_0 + n_i),$
  - example
    - $I = 2, n_0 = n_1 = 1, n_2 = 9$
    - link 2:  $x_2 = C / (1 + 9) = 0.1 C$
    - link 1:  $x_1 = C / (1 + 1) = 0.5 C$
    - $x_0 = \min (0.5 C, 0.1 C) = 0.1 C$
- Allocating equal shares is not a good solution
  - some flows can get more

## Example

- Problem
  - link 1 :  $0.6 C$ 
    - underutilized
  - link 2 :  $1 C$

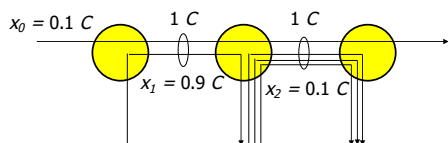


15

16

## Max-Min Fairness

- We can increase  $x_1$  without penalty for other flows
  - $x_0 = 0.1 C, x_1 = 0.9 C, x_2 = 0.1 C$



17

## Max-Min Fairness

- Allocating resources in an equal proportion is not a good solution since some sources can get more than others without decreasing others' shares
- Max-Min fair allocation
  - Min: because of the fairness on bottleneck links
  - Max: because we can increase throughput whenever possible

18

## Progressive filling

- Bottleneck link  $l$  for source  $s$ 
  - link  $l$  is saturated :  $\sum x_j = C$
  - source  $s$  on link  $l$  has the maximum rate among all sources using that link
- Progressive filling allocation
  - $x_j = 0$
  - increase  $x_j$  equally until  $\sum x_j = C$
  - rates for the sources that use this link are not increased any more
    - all the sources that do not increase have a bottleneck link (Min)
  - continue increasing the rates for other sources (Max)

19

## Example

- Parking lot scenario
  - $x_j = 0$
  - $x_j = d$  until  $n_0 x_0 + n_j x_j \leq C$
  - bottleneck link for  $d_i = \min(C / (n_0 + n_j))$ , source 0 or  $i$ 
    - $x_0 = \min(C / (n_0 + n_j))$
  - increase other sources
    - $x_j = (C - n_0 x_0) / n_j$
- In our example
  - $x_0 = 0.1 C, x_2 = 0.1 C$
  - $x_1 = 0.9 C$

20

## Proportional Fairness

- Equal share fairness and Max-min fairness
  - per link only
  - do not take into account the number of links used by a flow
  - flows  $x_j$  benefit from more network resources than flows  $x_i$
- Another fairness
  - give higher throughput to flows that use less resources
  - give smaller throughput to flows that use more resources
- Proportional fairness

21

## Proportional Fairness

- An allocation of rates  $x_s$  is *proportionally fair* if and only if, for any other feasible allocation  $y_s$  we have ( $S$  sources)

$$\sum_{s=1}^S \frac{y_s - x_s}{x_s} \leq 0$$

- Any change in the allocation must have a negative average change
- Parking lot example with  $n_s = 1$ 
  - max-min fair allocation  $x_s = C/2$  for all  $s$
  - let decrease  $x_0$  by  $\delta$  :  $y_0 = C/2 - \delta, y_s = C/2 + \delta, s = 1, \dots, I$
  - average rate of change is positive - not proportionally fair for  $I \geq 2$

$$\left( \sum_{s=1}^I \frac{2\delta}{c} \right) - \frac{2\delta}{c} = \frac{2(I-1)\delta}{c}$$

22

## Proportional Fairness

- There exists one unique proportionally fair allocation. It is obtained by maximizing

$$J(\vec{x}) = \sum_s \ln(x_s)$$

over the set of feasible allocations for all sources  $s$

23

## Parking lot example

- For any choice of  $x_0$  we should set  $x_j$  such that
  - $n_0 x_0 + n_j x_j = C, i = 1, \dots, I$

- Maximize
 
$$f(x_0) = n_0 \ln(x_0) + \sum_{i=1}^I n_i (\ln(C - n_0 x_0) - \ln(n_i))$$
 over the set  $0 \leq x_0 \leq C / n_0$ .

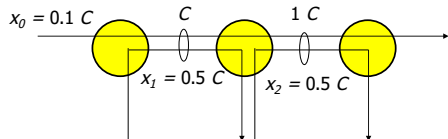
- The maximum is for
 
$$x_0 = \frac{C}{\sum_{i=0}^I n_i} \quad x_i = \frac{C - n_0 x_0}{n_i}$$

- If  $n_i = 1, x_0 = C/(I+1), x_j = C/(I+1)$
- Max-min allocation is  $C/2$  for all rates - sources of type 0 get a smaller rate, since they use more network resources

24

## Comparisons

- $I = 2, n_i = 1$
- max throughput:
  - $x_0 = 0$ , throughput =  $2C$
- equal-share and max-min:
  - $x_0 = C/2, x_1 = C/2$ , throughput =  $1.5C$
- proportional fairness:
  - $x_0 = C/3, x_1 = 2C/3$ , throughput =  $5C/3$



25

## End-to-end congestion control

- End-to-end congestion control
  - binary feedback from the network: congestion or not
  - rate adaptation mechanism: decrease or increase
- Modeling
  - $I$  sources, rate  $x_i(t), i = 1, \dots, I$
  - link capacity:  $C$
  - discrete time, feedback cycle = one time unit
  - during one time cycle, the source rates are constant, and the network generates a binary feedback signal  $\gamma(t) \in \{0, 1\}$
  - sources: increase the rate if  $\gamma(t) = 0$  and decrease if  $\gamma(t) = 1$
  - feedback

$$\gamma(t) = [\text{if } (\sum_{i=1}^I x_i(t) \leq c) \text{ then } 0 \text{ else } 1]$$

26

## Linear adaptation algorithm

- Find constants  $u_0, u_1, v_0, v_1$ , such that

$$x_i(t+1) = u_{\gamma(t)} x_i(t) + v_{\gamma(t)}$$

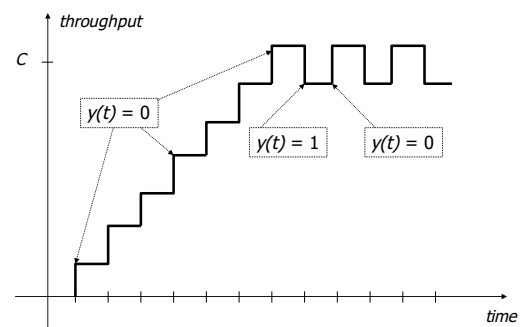
- we want to converge towards a fair allocation
- one single bottleneck, so all fairness criteria are equivalent
- we should have  $x_i = C/I$
- the total throughput

$$f(t) = \sum_{i=1}^I x_i(t)$$

should oscillate around  $C$ : it should remain below  $C$  until it exceeds it once, then return below  $C$

27

## Linear adaptation algorithm



28

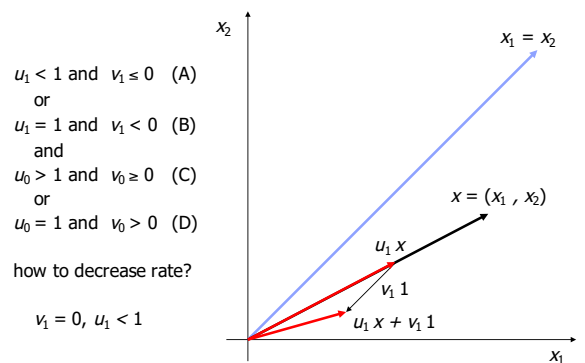
## Necessary conditions

$$f(t+1) = u_{\gamma(t)} f(t) + v_{\gamma(t)}$$

- we must have
  - $u_0 f + v_0 > f$ , increase rate if feedback 0
  - $u_1 f + v_1 < f$ , decrease rate if feedback 1
- this gives the following conditions
  - (A)  $u_1 < 1$  and  $v_1 \leq 0$
  - or
  - (B)  $u_1 = 1$  and  $v_1 < 0$
  - and
  - (C)  $u_0 > 1$  and  $v_0 \geq 0$
  - or
  - (D)  $u_0 = 1$  and  $v_0 > 0$

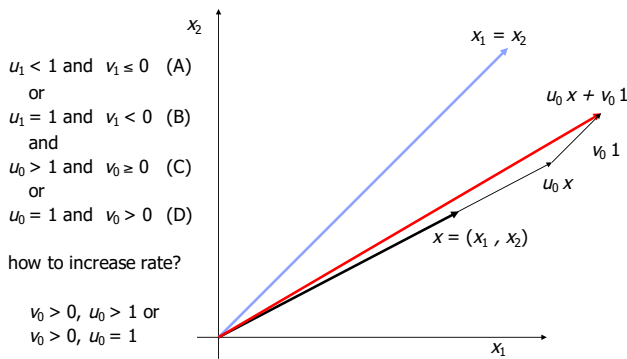
29

## Ensure fairness



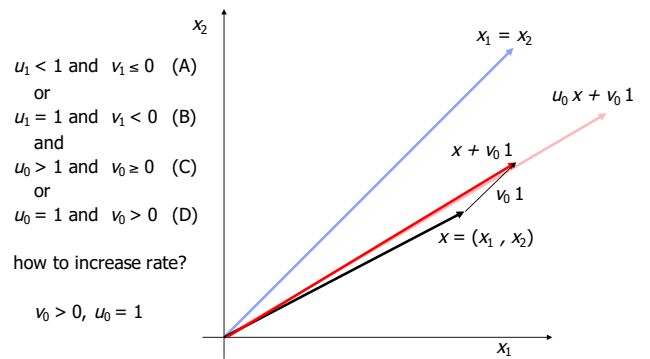
30

## Ensure fairness



31

## Ensure fairness



32

## Ensure fairness

- When we apply a multiplicative increase or decrease, the unfairness is unchanged
- An additive increase decreases the unfairness, whereas an additive decrease increases the unfairness
- To obtain that unfairness decreases or remains the same, and such that in the long term it decreases
  - $v_1 = 0$  decrease must be **multiplicative**
  - $u_0 = 1$  increase must be **additive**

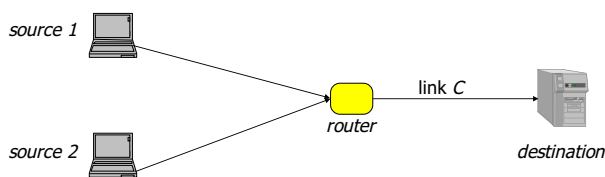
33

## Result

- Fact
  - In order to satisfy efficiency and convergence to fairness, we must have a multiplicative decrease (namely,  $u_1 < 1$  and  $v_1 = 0$  and a non-zero additive component in the increase (namely,  $u_0 \geq 1$  and  $v_0 > 0$ ).
  - If we want to favour a rapid convergence towards fairness, then the increase should be additive only (namely,  $u_0 = 1$  and  $v_0 > 0$ ).
- Additive increase, Multiplicative decrease**

34

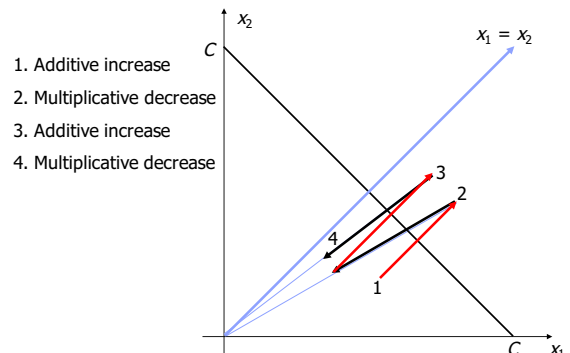
## Why AI-MD works?



- Simple scenario with two sources sharing a bottleneck link of capacity  $C$

35

## Throughput of sources



36

## Different types of CC

- Router/Switch centric (ATM)
  - switch decides which packet transmit or discard
  - switch notifies the source at which rate it should send
- Open loop (ATM)
  - resource reservation
  - admission control
- Host centric (TCP)
  - host observes the network and adjust the rate
- Closed loop with feedback
  - information on congestion state
    - implicit - packet loss (TCP)
    - explicit (RTCP)

37

## Different types of CC

- Rate-based control
  - negotiated with network
  - adjusted if needed
  - ATM, RTP
- Window-based control
  - defines the volume of data to send
  - TCP
- Open loop implies
  - Router/Switch centric
  - rate-based control

38

## Facts to remember

- In a packet network, sources should limit their sending rate by taking into consideration the state of the network
- Maximizing network throughput as a primary objective may lead to large unfairness
- Objective of congestion control is to provide both efficiency and some form of fairness
- Fairness can be defined in various ways: equal share, max-min, proportional
- End-to-end congestion control in packet networks is based on binary feedback and the adaptation mechanism of additive increase, multiplicative decrease.

39